## Combinatorics

## B. Math. III

## Mid-Term Examination

Instructions: All questions carry ten marks.

- 1. Let  $\alpha, \beta$  be two real numbers. Determine when a square matrix whose diagonal entries are  $\alpha$  and off-diagonal entries are  $\beta$  is invertible.
- 2. Let X be the set of points of a  $2 (n^2 + n + 1, n + 1, 1)$  design D. Let B be a block of D. Prove that the induced design on  $X \setminus B$  is a  $2 - (n^2, n, 1)$  design.
- 3. Let  $\mathcal{O}$  be a suset of points of a projective plane of order n whose no three points are collinear. Prove that  $\mathcal{O}$  contains at most n+2 points and equality can hold only when n is an even number.
- 4. Let L be a Latin square of order k. Determine all numbers n such that L is a subsquare of a Latin square of order n.