# Combinatorics 

B. Math. III<br>Mid-Term Examination

Instructions: All questions carry ten marks.

1. Let $\alpha, \beta$ be two real numbers. Determine when a square matrix whose diagonal entries are $\alpha$ and off-diagonal entries are $\beta$ is invertible.
2. Let $X$ be the set of points of a $2-\left(n^{2}+n+1, n+1,1\right)$ design $D$. Let $B$ be a block of $D$. Prove that the induced design on $X \backslash B$ is a $2-\left(n^{2}, n, 1\right)$ design.
3. Let $\mathcal{O}$ be a suset of points of a projective plane of order $n$ whose no three points are collinear. Prove that $\mathcal{O}$ contains at most $n+2$ points and equality can hold only when $n$ is an even number.
4. Let $L$ be a Latin square of order $k$. Determine all numbers $n$ such that $L$ is a subsquare of a Latin square of order $n$.
